Interactive Parametric Tools for Structural Design

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Summary

This paper presents a concept for a novel typology of computational design tools which are able to bridge the gap between the intuitive "structural sketch" and sophisticated analysis software. Based on equilibrium solutions and graphic statics, a set of interactive parametric tools have been implemented as a prototype, within an existing computer-aided design system. These tools allow for the powerful synthesis between structural design approaches based on funicular forms and associative geometric modelling techniques.

Keywords: structural design; interactive tools; graphic statics; funicular forms; parametric modelling; form-finding.

1. Introduction

Graphic statics, a structural design and analysis method based on the means of vector calculus, descriptive geometry and drawing was developed in the second half of the 19th century by Karl Culmann. He considered drawing to be the *true* "language of the engineer", as opposed to analytical methods using numerical calculus [1]. The advantage of equilibrium-based graphical techniques is the visualization of structural relations using diagrams of forces. Visualization allows for an intuitive understanding of complex technical dependencies by "thinking in pictures" and is deeply related to the creative act in engineering [2]. Furthermore, the manual process of drawing a force diagram, using drafting tools, leads not only to visualization, but to an *embodied understanding* of structural problems by the designer.

In design education, the value of graphic statics and its ability to produce intuitive and direct understanding of force systems has been emphasized by Luigi Nervi [3]. Several leading technical universities worldwide use graphic statics in teaching structures for architects, among these the MIT, Cambridge, USA [4], the RWTH Aachen, Germany [5], ETH Zurich [6] and EPFL Lausanne [7] in Switzerland. Recently, different web-based learning environments and teaching tools for structural design using graphic statics have been implemented [8, 9].

In design practice, famous designers such as Koechlin, Maillart, and Gaudi used graphic methods in order to guide design decisions [10]. Today, approaches based on equilibrium solutions such as strut-and-tie models [11] or stress fields [12] are used in the design of reinforced concrete structures. The main advantage of these methods is the ability to visualize a state of equilibrium and relate this state to the construction of the structure. Graphic statics and related methods are used in the assessment of the safety of historic masonry [13]. The contemporary Swiss engineer Conzett used graphic statics, to design the geometry of a cable bridge [14].

In this paper, the basic concept for a novel typology of computational structural design tools is presented, together with a prototypical implementation of the core functionalities. The aim of these computational tools is to bridge the gap between the "structural sketch" and sophisticated analysis software. Using a vector-based technique derived from graphic statics, the presented tools allow for a new powerful combination of associative modelling methods and structural optimization approaches based on funicular forms, similar to a recently proposed system [15]. Tight integration in the contemporary design workflow, visualization and interactive feedback in real time will lead to an intuitive understanding of structural correlations and thereby advance creativity in the design process. The conceptual framework for the integration of structural constraints in a parametric model has been described earlier [16]. The presented tools now offer a systematic approach for the integration of structural constraints into parametric systems in general.

The content of this paper is as follows. Section 2 briefly summarizes the theoretical background of the tools: equilibrium solutions in structural design, graphic statics, and associative modelling techniques. Section 3 describes basic concepts and prototypical implementation of the presented tools. To conclude, section 4 shows two applications: a parametric 3-dimensional truss, as a design example, and the analysis of an arch with two fixed ends.

2. Theoretical background

2.1 Equilibrium solutions

The structural design approach presented in this paper, based on equilibrium solutions, is predicated on the lower bound theorem of the theory of plasticity and the rigid-plastic material model [17]. These assumptions allow for describing the transfer of loads by only considering equilibrium of forces while neglecting material stiffness and deflections. All systems modelled with the tools presented here are considered as statically determinate or kinematic pin-jointed structures, with axial forces in the members only. This basic design approach allows for direct assignment of minimum dimension for all structural members for a given state of equilibrium. For these member dimensions, the structure is safe according to the lower bound theorem of plasticity theory, as long as no elements are in danger of buckling. Plasticity theory has been applied successfully to the design of steel [18], reinforced concrete [12] and masonry [19]. In general, the theory can be applied to any ductile material. The designer has to use additional means for stiffening, if the system forms a mechanism, in order to prevent it from collapse for asymmetric load cases.

2.2 Graphic statics

Graphic statics can be described as a set of geometric algorithms based on vector calculus and descriptive geometry in plane [20]. These algorithms allow for solving problems from vector calculus, as they occur in the search for equilibrium solutions. Forces are represented by position vectors; the geometry of a structure is represented by a line diagram of the axes of all structural members and external loads (Fig. 1). This diagram is called the *form diagram*, while the force distribution in the structure is represented by the *force diagram*. Each axis in the structure is corresponds to one parallel line in the force diagram; the line's magnitude is proportional to the inner force in the element. Each node in the form diagram corresponds to a closed polygon in the force diagram. Global equilibrium is guaranteed through a closed diagram. The geometric relationship between form and force diagram is called *reciprocal*. For a statically determinate structure, there is one unique force diagram, irrespective of scaling. The power of the graphical method is related to its ability either to construct the force diagram from the form diagram, thus using graphical techniques for analysis purposes, or to construct the form diagram from the force diagram, in this way applying a form-finding algorithm. For this paper, the focus is clearly on the latter approach.

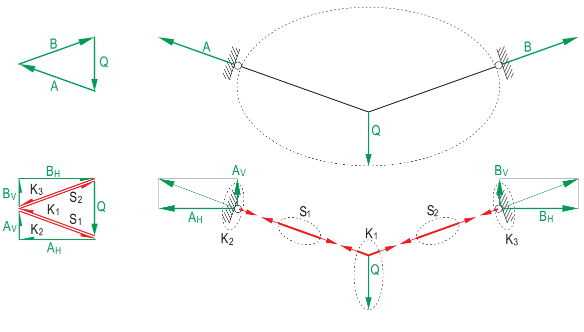

Fig. 1: Form and force diagram of a hanging cable for one node (above); joined diagrams for three nodes (below); from [6]*, p. 17*

2.3 Associative modelling

In the last twenty years, influenced by software developments in the airplane and animation film industries, parametric modelling environments have become increasingly popular among architects and engineers [21]. Programmable interfaces enable the designer to automatically generate geometric objects according to a self defined algorithm. By these means, the designers gains full control over modelling and construction of complex forms. In the last decade, a new evolution of software has allowed for the definition of geometric relations between objects of 3-dimensional CAD models using visual programming editors (VPE). These editors visualize relations between objects, and thus lead to an intuitive programming practice. Moreover, such systems enable the designer to inscribe certain constraints in the geometry, with the result that one part of the model is depending on the geometry of another part. The relations between geometric objects are mainly described with the means of vector calculus; a *parametric definition* basically describes a geometric algorithm.

3. Parametric tools for structural design

3.1 Implementation

The core functionalities of the tools presented in this paper has been implemented prototypically for the computer-aided design (CAD) software *Rhinoceros 4.0* together with the parametric modelling plug-in *Grasshopper*. This plug-in uses a VPE as an interface to the user, enabling the definition of parametric relations between geometric objects and numerical parameters. Each parametric definition consists of functional units, the *components*, which are linked with "wires" that transfer data (Fig 2.). A component processes input data that is received from the nodes on the left side and produces resulting data that is transmitted from the output nodes on the right side of the component. Data can be a list of numbers, vectors, or objects. Special components are used to link object from the CAD modelling environment into the VPE. A variety of functions for geometric operations numerical calculus, and vector calculus, are already available as components. Additionally, there is the possibility to create user-defined components using the scripting language VB.NET. This facility has been used in order to add the tools described below to the associative modelling system. The following paragraphs will describe some core concepts in detail.

3.2 Forces

In the proposed system, a force is represented by a straight line positioned in space; the direction of the force is defined by its internal parameterization, resulting from the drawing sequence of start and end point. The magnitude is given by the line length. Any straight line will be interpreted as a force, if connected with the corresponding input nodes of a component that interprets the line as force. Visualization of external forces is provided by a component that draws a green arrow at the position of the input line, in the direction of the force. The inner force in a member of a structural system is described as the couple of a real number and a line object that defines the member's axis. The sign of the number identifies whether the element is in tension or compression. Inner forces are represented by cylinders around the element's axis; its diameter is defined by the magnitude of the inner force, tensions forces are coloured in red, compression forces in blue.

3.3 Systems

With the tools presented in this section the user is able to create basic pin-jointed planar systems that are determinate or kinematic. The concepts of graphic statics are used, to understand the degrees of freedom in these systems and to solve them. Structural systems are formed by the interpretation of lines, curves and points as structural elements and forces, through custom components. Therefore, three simple examples are discussed (Fig. 2-4). The first two examples present a kinematic and a determinate system. The third example shows a structural system, that combines the two antecedent ones.

In general, for a fixed geometry and an arbitrary set of forces, a kinematic system, as it is illustrated in figure 2, has no state of equilibrium. Graphic statics provides a strategy to re-formulate the problem in a way, that it has a unique solution, and thus becomes "determinate". The construction of the *funicular* used in graphic statics is a method to solve the problem [22]. Funicular forms represent the shape of a hanging cable or arch for a given loading. By releasing the geometric constraints, if the joints of the structure are not fixed to a point, but may slide along a straight line defined by the force, the system becomes uniquely solvable for a chosen *rise*. The rise is a numerical parameter that "scales" the solution along the force lines, called *lines of action*. The force diagram has the typical fan structure, consisting of small triangles, each representing one node of the funicular in equilibrium. With a change of the input parameters, supports, external forces or rise, the geometry of the funicular polygon and the force diagram updates in real time. To solve the problem algorithmically, the method mentioned above is implemented by means of vector calculus. The concept of the line of action allows a highly controlled form-finding approach to cables and arches.

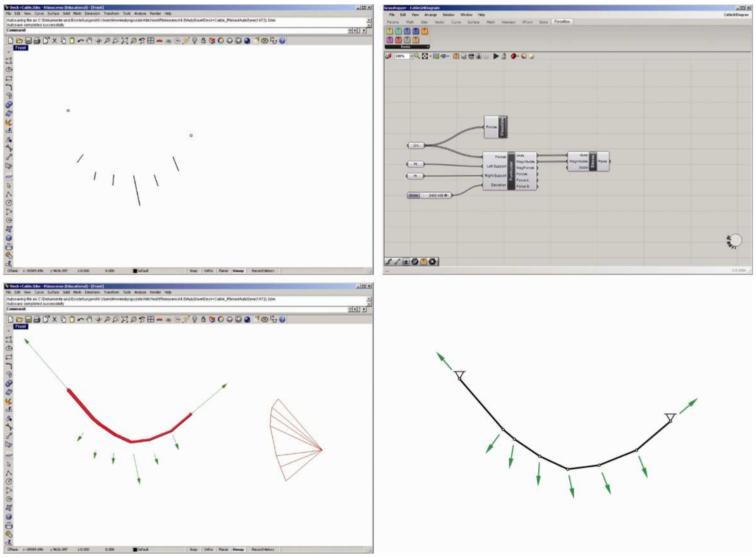

Fig. 2: The funicular component. Input: lines as forces and points as supports, and rise as real number (left above). Parametric definition: Funicular component and two components for visualization of inner and external force (right above). Output: funicular polygon, force diagram, reaction forces (left below). Structural system: kinematic (right below).

A statically determinate structural system, as shown in figure 3, has a unique solution. Unlike the funicular, this system is directly solvable for any given geometry and loading. The idea of that component is the analogy of a cantilevering bridge deck with inclined hangers for given loads in the z-direction at all joints. The component solves for inner forces and support reactions. The solving method is based on the means of vector calculus, starting at one end of the deck. At the outer left joint, the nodal loading l_1 can be decomposed into two forces, one in the direction of the first deck segment, the other in the direction of the first hanger $l_1 = h_1 + s_1$ in a unique way. This node is represented by the triangle at the upper tip of the force diagram. The next node can be solved in a similar manner: nodal loading l_2 and the force in the deck s_1 result in the force $r_1 = l_2 + s_1$. The resultant r_1 is decomposed in the forces in direction of the second hanger and the second deck component $r_1 = h_2 + s_2$ uniquely, and so forth. This node is represented in the force diagram by the quadrilateral below the triangle. At the right end of the system, the force s_n in the direction of the expansive bearing remains.

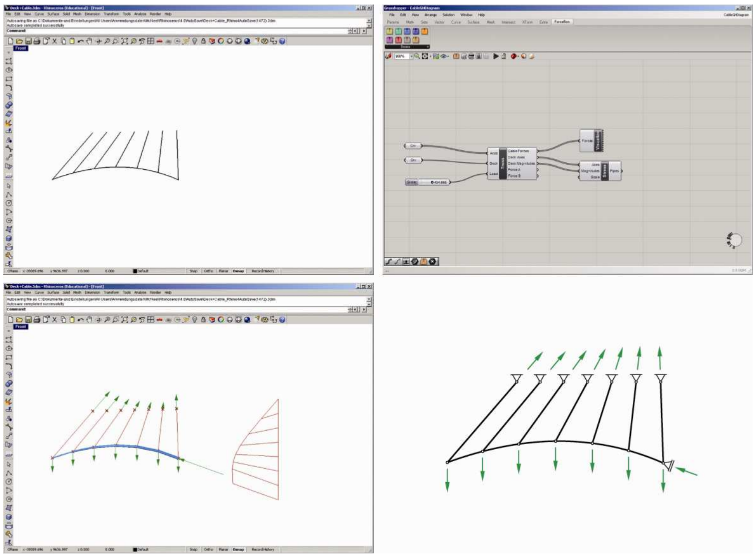

Fig. 3: The cantilever component. Input: curve as deck, variable number of lines as force axes, numerical load magnitude (left above). Parametric definition: cantilever component, and two components for visualization of inner and external force (right above). Output: force diagram, reaction forces (left below). Structural system: determinate (right below).

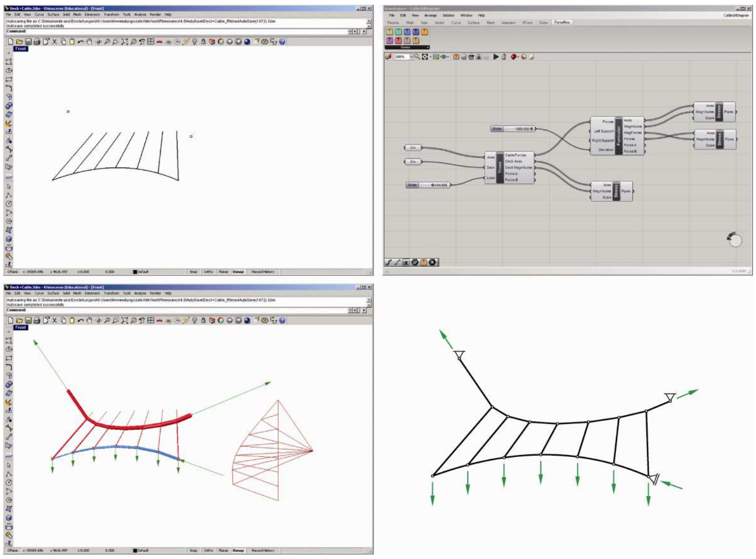

Fig. 4: *The system combined from the cantilever and the funicular system: Input: deck curve, hanger axes, supports of the funicular as points (left above). Parametric definition: merged definition of the cantilever and the funicular (right above). Output: reaction forces and joined force diagrams (left below). Structural system: kinematic (right below).*

The system shown in figure 4 is a combination of the two antecedent systems: the cantilevering deck is combined with a funicular. The forces pulling from the deck of the cantilever downwards are transferred into the hanger. By connecting the output node of the cantilever component to the input node of the forces of the funicular in the visual programming editor, the two systems merge. Also, the two force diagrams join together in the lines that represent the hanger forces, respectively the external forces of the funicular system. This example clearly illustrates the advantage of using a visual programming editor. The combination of structural sub-systems in greater structures becomes very intuitive through the visualization of both the geometry and the functional relationship defined by the component network in the VPE.

3.4 Data transition

To provide a smooth transition of the parametric model data to commercial structural design software, in order to carry out a analysis of the usability of the designed structure in later design stages, a custom routine has been developed using *Rhinoscript*, a scripting language for the CAD system Rhinoceros. This routine is able to export supports, axis and external forces in a structured branches/nodes table, in this case tailored for the FEM software *SOFiSTiK Version 24*. For this paper, member sections are generated in proportional relation to the inner force. An implementation that would incorporate also buckling of the members is straight forward.

4. Results

4.1 Analysis of an indeterminate arch

In section 3, exclusively pin-jointed determinate or kinematic systems have been modelled. This example shows how to use the proposed tools combined with a generic minimization routine, for the analysis of a planar indeterminate structure. An arch with two fixed ends is analysed using the concept of the *elastic line of thrust* devloped by Winkler [23]. In order to find the "correct" line of thrust from the infite number of possible funiculars, he assumed that the one closest to the center line of gravity, in a least square sense, is the solution. Therefore, Winkler analytically solved the minimization of the sum of squares *I* of the vertical distances from the thrust line to the line of gravity, in dependency of three free parameters of the arch.

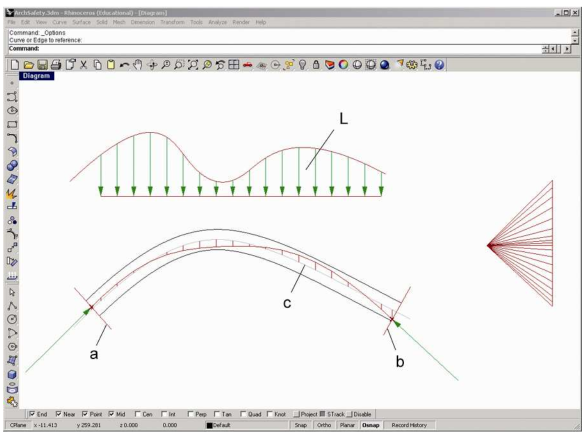

Fig. 5: *Elastic line of thrust for a given arch and loading*

For this example, a distributed Load *L* defined by two curves, the arch geometry including the line of gravity *c*, and two curves *a, b* defining the two supports, is given (Fig. 5). The distributed load *L* is represented by a discrete set of forces. A parametric model of the funicular *f* defined by the load *L*, the supports $A = a(p)$, $B = b(q)$ as parametric description of the input lines a and b, and rise r, is set up. The parameters p and q are identifying the position of the support points on the lines a and b. The vertical distances between f and c are referred to as z_i. The elastic line of thrust is uniquely defined by the combination p_{el}, q_{el}, r_{el}, which minimizes (1) for $p, q \in [0; 1]$, $r > 0$.

$$I = \Sigma z_i^2 \qquad (1)$$

A generic minimization tool that is a build-in part of the associative modelling system is used, in order to minimize I by a parameter search for p, q and r. The distance of the elastic line of thrust to the arch edges is a measure for the safety of the arch. The application of this elastic arch theory is only valid for fixed supports, a condition that most likely is not the case for historic masonry arches, but might be the case for reinforced concrete structures.

On the one hand, the built-in minimization routine is not very powerful; the solving time takes up to 30 seconds for the presented example with 18 single loads, using an Intel Core Duo Processor with 2.8 GHz. On the other hand, the used minimization component, based on a genetic algorithm, is very flexible and robust; it can be used for the parameter search of any value that is described in a parametric relationship from input values. Similar approaches could be used to solve other indeterminate structures.

4.2 Design of a parametric funicular system in space

The examples presented in section 3 are all planar systems. This example will show a combination of several planar systems to a spatial system in one parametric model. The aim of this model is to generate a trussed structure with efficient load-bearing capacities for a given set of loads applied to one chord that has a given arbitrary shape in elevation. The model is controlled by several geometric and a numerical parameter, the form of the funicular chord adapts to parameter changes. The structure is symmetric in plan.

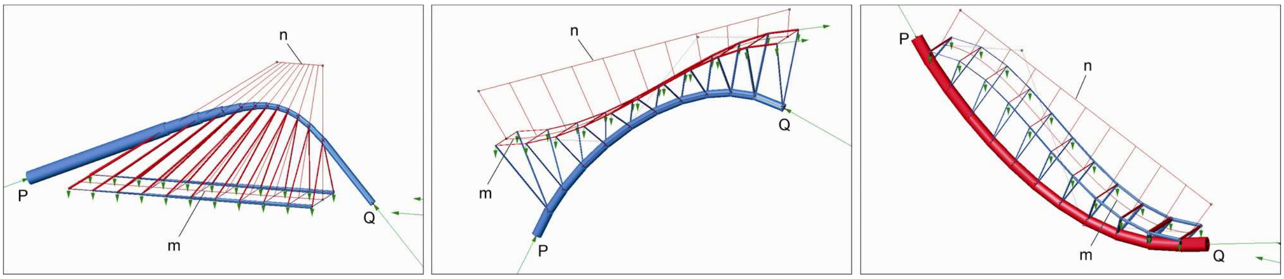

Fig. 6: Three instance of the parametric model of a funicular structure in space. The first example (left) is based on a straight line m *defining the elevation of the loaded chords, a rise* $r > 0$ *that results in a funicular in compression, and the axes between* m *and* n *defining the position and orientation of the connecting members between the chords. For the second example, the supports* P *and* Q *are lowered, and* m *is replaced by a curve, still the rise* $r > 0$, *position and orientation of* n *is adapted. In the third example (right),* m *is deformed, the position of the supports is lifted, and the value of the rise* $r < 0$, *thus the funicular is in tension.*

The geometric parameters are two support points of the funicular P and Q, a guiding curve m of the elevation of the two load-bearing chords as spline, and a straight line n that controls the directions of the truss members connecting both load-bearing chords with the funicular (Fig. 6). Numerically, the number of connecting members, the width of the upper chord, and the rise of the funicular can be controlled. The curves m and n are divided in segments, the division points are connected by straight lines. These lines determine the axes of the connecting members between the upper and lower chord. The geometric input objects, P, Q, m and n have to lie in one vertical plane E. This parametric model is based on the system presented above (Fig. 4). Basically, the loaded chord has been split up into two chords that are symmetrically positioned on both sides of E. The triangles that connect the funicular with the loaded chord are separate planar systems. This system has four support reactions: two forces at the supports of the funicular, and two forces at the end of the upper deck.

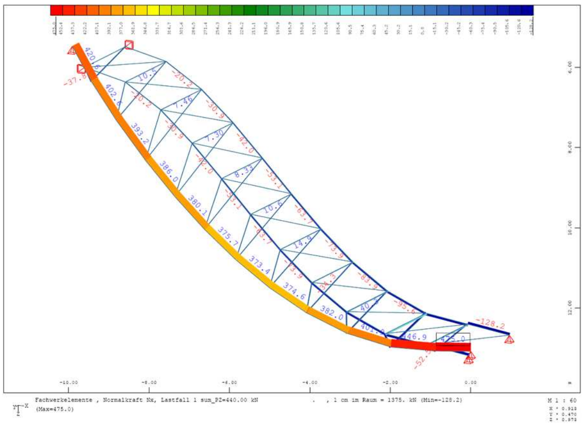

Fig. 7: Result of the FEM analysis, based truss elements that can only transfer axial forces

This parametric model allows for the intuitive exploration of the formal freedom of the structure in space. The visualization of magnitude and sign of the axial forces leads to an intuitive understanding of the effect of geometric transformations on the inner force-flow. Due to the funicular shape of one chord, in general, the force-flow of all instances of this model is highly efficient. Changes in the controlling parameters of this model with 10 segments are updated in approximately one second, using an Intel Core Duo Processor with 2.8 GHz. That seems to be slow, but the prototypical character of the implementation of the tools via a scripting language has to be taken ino account.

In order to verify the proposition of equilibrium of the parametric geometry for a set of external forces, one instance of the structure, including the loads, has been transferred to the FEM analysis software SOFiSTiK, via the tailored interface described in section 3.4. A finite element analysis, using truss elements that can only transfer axial forces, has been carried out. The first naive approach failed. The used solver based on the Crisfield method could not find an equilibrium state with the supports provided in the parametric model; it failed by torsion of the upper chords. Several means had to be introduced to avoid this: additional vertical sliding supports at the upper ends of the upper chords have been introduced, as well as diagonals between these chords. Finally, for the system shown in figure 7, which is still kinematic, the predicted equilibrium state could be reproduced using the analysis software.

5. Conclusions and Future Work

The results presented in the previous section demonstrate the power and validity of the described concepts, and the flexibility of the framework for design and analysis even with only a few prototypically implemented functionalities. The parametric model from section 4.2 shows the possible divergence in geometry and inner force-flow of different instances of a parametric model based on these tools: different structures can be derived from the same model with just a few parameter changes, while maintaining the structural efficiency resulting from the funicular shape of the chord. The arch analysis described in section 4.1 illustrates the value of the presented tools as a flexible analysis environment. Besides this, the didactical significance of this component-based attempt should be emphasised: the possibility to build up advanced structural analysis concepts from basic ones. Finally, the visual and interactive approach presented in this paper helps to bridge the gap between design and analysis methods, and might advance creativity in early stages of structural design.

Future work will focus on the extension of the concepts presented in this paper to fully 3-dimensional funicular forms. A publication about concepts for the generation of linear funiculars in space is planed [24]. Furthermore, tools for the design of spatial planar structures like shells and space frames will be incorporated in the framework.

6. Acknowledgements

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